TRINITY COLLEGE

Year 12 Mathematics Methods
Test 6 2020

Calculator Assumed

Point Estimates and Confidence Intervals

STUDENT'S NAME SOLUTIONS- MALLIS

DATE: Monday 7th September **TIME:** 45 minutes **MARKS**: 43

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (7 marks)

One normal fair dice is rolled, and a coin is flipped. This is repeated a further 199 times and the 200 trials produce a head with a number less than 5 on 74 of the 200 occasions.

(a) What is the value of p, the population proportion of obtaining a head with a number less than 5 when the dice is rolled, and coin is flipped. [2]

$p = \frac{4}{12}$

$= \frac{1}{3}$

	H	T
1	H1	T1
2	H2	T2
3	H3	T3
4	H4	T4
5	H5	T5
6	H6	T6

(b) What is the value of $\hat{p}$, the sample proportion of a head with a number less than 5 for the 200 trials mentioned? [1]

$\hat{p} = \frac{74}{200}$

(c) Calculate the mean and standard deviation of $\hat{p}$, the sample proportions for samples of 200 trials. [2]

$\mu_{\hat{p}} = p$

$= \frac{1}{3}$

$sd = \sqrt{\frac{\frac{1}{3}(1-\frac{1}{3})}{200}}$

$= \frac{1}{30}$

(d) How many standard deviations from p is our value for $\hat{p}$? [2]

$z = \frac{\frac{74}{200} - \frac{1}{3}}{\frac{1}{30}}$

$= 1.1$

2. (8 marks)

It is known that at a certain school, the proportion of students who play basketball is p. A random sample of 150 students was selected and of these students 112 played basketball.

(a) Determine a point estimate for the proportion of students who play basketball. [1]

$$\hat{p} = \frac{112}{150}$$

(b) Determine a 90% confidence interval for p. [2]

$$0.688 \leq p < 0.80$$

(c) A second sample of 150 students was taken. In this second sample the number of students found to play basketball was 101. Use the confidence interval from (b) to determine if the students in the second sample are statistically different to those of the first sample. [3]

$$\hat{p} = \frac{101}{150} \qquad 0.6103 \leq p \leq 0.7363$$

Samples overlap so suggests they are <u>not</u> statistically different

(d) In a third sample of 150 students, the 99% confidence interval for p was $0.618 \leq p \leq 0.808$. How many students in this sample play basketball? [2]

$$\hat{p} = \frac{0.618 + 0.808}{2} = 0.713$$

$$n.p = 150 \times 0.713$$

$$= 107 \text{ students}$$

3. (7 marks)

In a sample of n houses, 27 were found to have smoke detectors. Using this sample, a c% confidence interval for the true proportion of houses with smoke detectors was $0.26 \le p \le 0.46$.

(a) Calculate the value of n. [2]

$$p = \frac{0.26 + 0.46}{2}$$

$$= 0.36$$

$$\frac{27}{n} = 0.36$$

$$n = 75$$

(b) Calculate the value of c. [5]

$$0.46 = 0.36 + z\sqrt{\frac{0.36(1-0.36)}{75}}$$

$$z = 1.8042$$

z score for CI

$$c = 0.9288$$

$$\therefore c = 93\%$$

4. (9 marks)

A recent poll commissioned by prominent Perth business figures, sampled 10 478 voters asking them the following question:

Who of the following do you think would make the better Premier?

4574 of the respondents indicated they preferred Mark McGowan over Liza Harvey.

(a) Population figures from the Australian Bureau of Statistics currently state that WA has a population of 2.72 million people. If the 99% confidence interval was given as $0.4241 < p < 0.4490$ determine the minimum number of people (to the nearest 100 000) who think Mark McGowan would be a better Premier. [2]

$= 0.4241 \times 2.72$

$= 1.153552$

$\approx 1200\,000$

(b) Using $\hat{p}$ as an estimate for the true proportion of WA voters who prefer Mark McGowan, determine the minimum number of people who need to be sampled to achieve a margin of error, at most, 0.01 for a 99% confidence interval. Show full working. [4]

$$\hat{p} = \frac{4574}{10478} = 0.4365$$

$$0.01 = 2.576\sqrt{\frac{0.4365(1-0.4365)}{n}}$$

$$n = 16320$$

(c) The original 10478 voters were also asked the following question:

Would you be more or less likely to vote for the Labor party if Mark McGowan were replaced as leader?

Without knowing how many people indicated this, determine the maximum possible margin of error for a 99% confidence interval. [3]

let $\hat{p} = 0.5$

$$E = 2.576\sqrt{\frac{0.5(1-0.5)}{10478}}$$

$$= 0.0126$$

5. (4 marks)

It is known that p % of students in a certain state of Australia are international students. 50 samples of 100 students are taken and the proportions of international students were calculated. The sampling distribution of the sample proportions has a standard deviation of 0.035. Determine with reasons, a reasonable value of p.

$$0.035 = \sqrt{\frac{p(1-p)}{100}}$$

$$p = 0.1429 \quad \text{OR} \quad 0.8571$$

0.14 is the reasonable values as we expect a lower proportion of international students.

6. (4 marks)

28% of orange trees never bear fruit. A wholesaler purchases 126 trees. Use sample proportion techniques to determine the probability that at least one third of these trees will not bear fruit. Show any distributions used.

$$p = 0.28 \qquad n = 126$$

$$sd = \sqrt{\frac{0.28(1-0.28)}{126}}$$

$$= 0.04$$

$$X \sim N(0.28, 0.04^2)$$

$$P(X > 1/3) = 0.0912$$

7. (4 marks)

When taking samples of size 400 from a population, it was found that 6% of samples had a proportion that was more than 0.03 above the population proportion. Determine the population proportion.

z score for $P(Z > k) = 0.06$

$k = 1.5548$

Using standard normal distribution

$$1.5548 = \frac{\hat{p} - p}{\sqrt{\frac{p(1-p)}{400}}}$$

$$1.5548 = \frac{0.03}{\sqrt{\frac{p(1-p)}{400}}}$$

$p = 0.182$ OR $p = 0.818$